Annex A

ADVECTION TESTS

Performance of the model advection scheme was examined in the classical numerical tests: rotational flow (called also as ‘solid body rotation’) and strong deformational flow [*Smolarkiewicz*, 1982]. The standard version of the model was applied in the tests. For this purpose the appropriate artificial wind field was used for the calculations and all emissions were switched off. Since both tests simulate non-divergent flow the surface pressure was set to a constant. Besides, the model orograthy was flattened. The initial distribution of a substance mixing ratio had a shape of three-dimensional ellipsoid with the circular cross section in the horizontal. The centre of the distribution was located at the fourth model layer. The substance mixing ratio is decreased linearly form the ellipsoid centre to some background value at its boundaries. An important issue of the tests is non-zero background mixing ratio, which allows testing the scheme positive definiteness and monotonicity. The three-dimensional advection was modelled under the defined conditions for a certain time interval and the resulting distribution was analysed. The main difference of the tests performed form the original ones is the model operation under the real conditions of the polar stereographic projection instead of flat Cartesian geometry. The wind fields were first defined on the Earth’s surface and then used for advection modelling on the projection. Besides, three-dimensional character of the advection leads to more significant artificial diffusion in comparison with original two-dimensional tests of the scheme [*Bott*, 1992].

Rotational flow

The objective of the rotational flow test is to examine the model ability to simulate a pollutant horizontal advective transport and to evaluate artificial diffusion of the numerical scheme. The wind field of the rotational flow is illustrated in Fig. A.1 by circular isolines of large radius. The axis of the rotational flow is located at the zero meridian and sloped down from the Earth axis with the angle 30°. The center of the initial distribution with the radius of 5 gridcells is located at the cell (25,25). Since the flow is non-divergent the uniform distribution should remain uniform, whereas the ellipsoid should theoretically be transported unchangeable. However, in reality it is smoothed down due to the artificial diffusion.

Results of the test are presented in Fig. A.1. The numbered sets of smaller concentric circles show the cone location and shape at different moments of its rotation. Fig. A.2 show the results in the form of the three-dimensional surface of mixing ratio distribution. As seen from the figures the initial distribution somewhat changes its shape due to the numerical dispersion and is smoothed down because of the artificial diffusion. The smoothing of the initial distribution after the whole rotation does not exceed 30% that is reasonable taking into account small radius of the initial distribution. Besides, insignificant non-monotonicity appeared by the end of the rotation. Thus, the advection scheme does not produce considerable distortions, has comparatively low artificial diffusion and almost monotone.

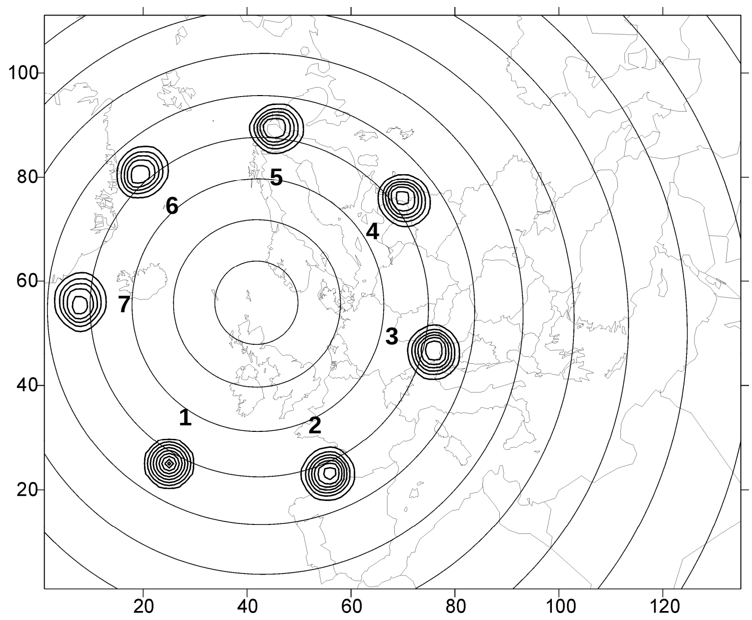

Fig. A.1. *Initial conditions and the results of the rotational flow test. Concentric circles of large radius denote wind velocity isolines. Smaller circles are the mixing ratio distribution isolines at the fourth model layer: 1 – initial; 2 – after 96 iterations; 3 – 192 iterations; 4 – 288 iterations; 5 – 384 iterations; 6 - 480 iterations; 7 - 576 iterations*

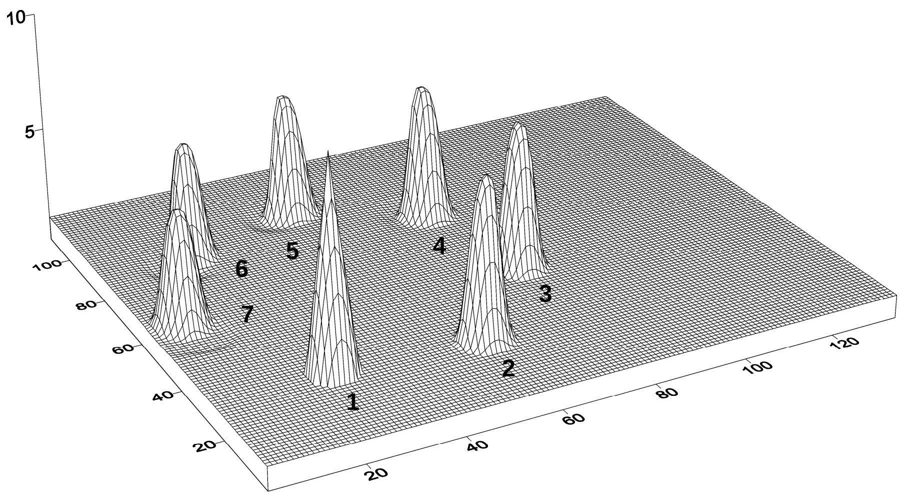

Fig. A.2. *The same as in Fig. A.1 but in the form of a three-dimensional surface of mixing ratio distribution*

Deformational flow

The objective of the deformational flow test is to examine the model stability in strong deformational flows and evaluate possible time-splitting error [*Bott*, 1993; *Easter*, 1993]. The velocity field for the deformational flow test and the initial mixing ratio distribution are presented in Fig. A.3. Zonal and meridional components of the wind are determined by the following formulas:

$$\begin{aligned} V_\lambda &= 4\sin(10\lambda)\sin(16\varphi), \\ V_\varphi \cos\varphi &= 2.5\cos(10\lambda)\cos(16\varphi), \end{aligned} \qquad \text{(A.1)}$$

where λ and φ are longitude and latitude of a gridcell, respectively.

The center of the initial distribution with the radius of 16 gridcells is located at the cell (64,63). As it is seen from the figure the wind field is built up from sets of symmetrical vortices. Since the flow is non-divergent again a uniform distribution should remain uniform except gridcells where mass from the initial ellipsoidal distribution incoming. But anyway mixing ratio values should remain limited at any point of the domain.

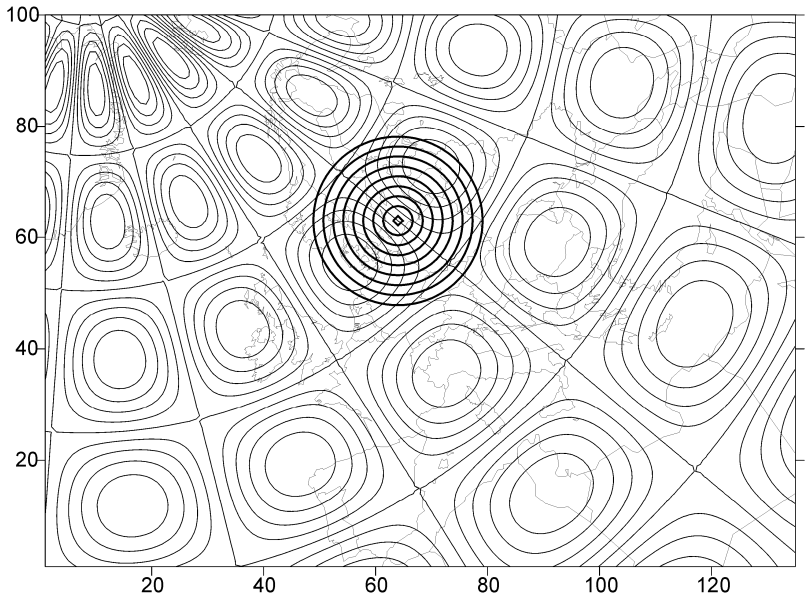

Fig. A.3. *Initial conditions of the deformational flow test. Cell-shaped curves denote wind velocity isolines. Circles show the initial mixing ratio distribution isolines at the fourth model layer*

Results of the experiment are presented in Fig. A.4. The figures illustrate transformation of the initialdistribution in the deformation flow and correspond to different time moments (or different numbers of iterations). As one can see the mass is coming along the boundaries of the vortices and is penetrating to the neighboring ones. The difference of this deformational flow test from the original one [*Smolarkiewicz*, 1982] is that boundaries of the vortices do not exactly coincide with gridcell borders. It leads to mass exchange between the vortices and smearing of the initial distribution over the domain. The maximum value of the distribution decreases in time until two stable elevations are formed. That means that the model is stable to the deformational flow and no observable time-splitting error appears.

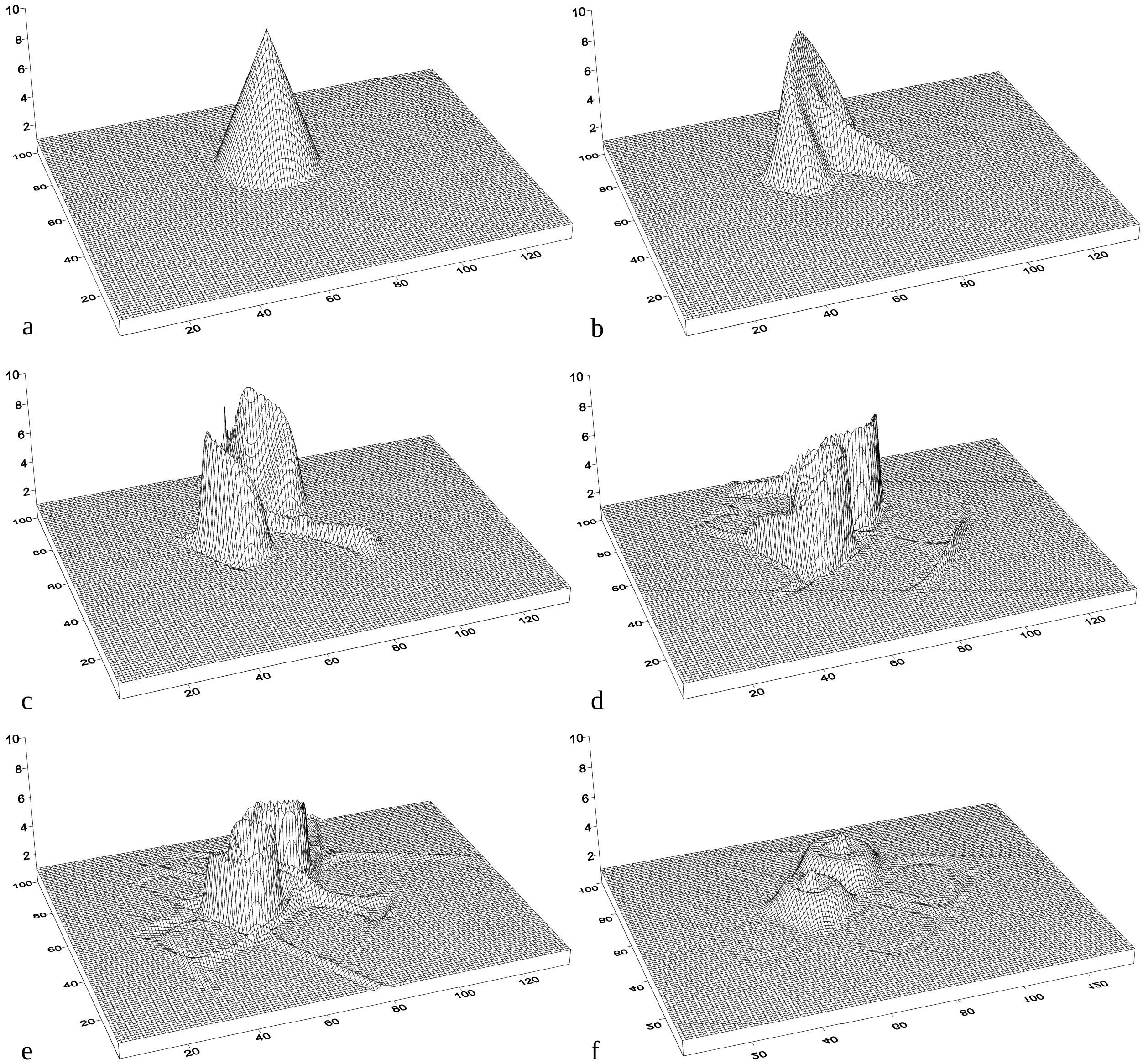

Fig. A.4. *Results of the deformational flow test. Three-dimensional surface of mixing ratio distribution at the fourth model layer: a – initial; b – after 48 iterations; c – 96 iterations; d – 192 iterations; e – 384 iterations; f - 1536 iterations*